

Profitability of the Pair Trading Strategy across Different Asset Classes

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Abstract

We perform an extensive empirical analysis of performance of pairs trading, a popular relative-value arbitrage strategy, based on four different selection methods—the Minimum Distance, Augmented Dickey Fuller Test and Granger Causality test, Linear Regression, and Correlated Remaining methods—across different asset classes including the Tehran Stock Exchange (TSE) shares, and components of S&P500 as well as commodities from February 2013 to May 2015. Results of the empirical test of four methods demonstrate that using different asset classes yields an excess return more than market. In addition, Minimum Distance can be considered the best method for application of the pairs trading strategy with an average annualized excess return of about 22%.

Keywords: Pair trading, Asset classes, Distance method, Augmented Dicky Fuller (ADF) test, Granger causality (CG) test

JEL Classification: C01, F21, G11, G15

1. Introduction

Since its birth in the 1980s, pairs trading have been popular as a statistical arbitrage strategy among major investment banks and hedge funds. Despite the high average annualized excess return, the idea behind the strategy is simple. This strategy is comprised of two stages. In the first stage (the formation period) the method applied to form pairs; and second (the trading period), the criteria for opening and closing positions. If the two prices of a pair of stocks move together in the past, they are likely to continue in the future. So when the prices diverge, a trader can simply take a short position with the over-priced stock and a long position with the under-priced one, and as effect of mean reversion, wait for the prices to converge in the future. When they do, the trader clears the positions and makes a profit (Narayan and Smyth 2007, Elliot et al. 2005).

The market neutral strategy normally provides protection against market risk by taking the long and short positions simultaneously regarding both assets to prevent exposure to direct risks. Hence, pairs trading can be considered a market neutral strategy. However, this statement does not suggest that

investment in pairs trading is risk-free or entails neutral risks. In any case, risks in these circumstances differ from risks directly posed to long investment.

Pair traders are exposed to model risks that are occurred when a strategy model does not meet expectations. Moreover, implementation risk is another factor that may negatively influence return of pairs trading which is attributed to the possibility that a strategy will not be executed as planned. Other than risks related to pairs trading, this investment technique imposes costs on traders, including the broker's commission. If an investor intends to buy easily, a commission for engaging in the trade and a commission for ending the trade are charged (Braten Nordby and Berg 2013). Pairs trading have been studied worldwide using various models. These studies have mostly investigated different methods of pair selection to analyze profitability. Gatev et al. (2006) studied profitability and performance of the pairs trading strategy using the minimum Distance method in selecting pairs and thresholding two standard deviations for pairs trading. They found that in spite of considering trading costs and risk factors, an annual return of 11% is achieved. Using this practice by Gatev which is the main and initial references for pair trading, we chose the Minimum Distance as one out of four methods in this study. Papadakis and Wysocki (2007) studied the effect of accounting information on profitability of this strategy. Results revealed that during accounting information events, pairs are more often opened than under normal circumstances. Moreover, pairs that are opened immediately following these events are less profitable than other pairs. Huck (2009) showed profitability of a model, which was proposed using the multiple prediction model and the multiple criteria decision making model, to obtain a ranking for stock selection and use the decision matrix and neutral network concept to model price spread. Baronyan et al. (2010) proposed a model for optimal selection of pairs by combining the Dicky Fuller (DF) method, two-way Granger Causality (CG) method, and Market Factor Ratio methods. In order to develop the work of Baronyan, the second method in this study is the combination of Augmented Dicky Fuller and Granger Causality tests. They applied the Vasicek model to a highly volatile crisis period and achieved considerable profitability. In an investigation, Do and Faff (2012) developed Gatev's work and indicated that although profitability of pairs trading has decreased in general, this strategy gave a brilliant performance during a period of recession (such as the 2008 financial crisis). The understudied period of our paper is a period of recession and with emphasize to the results of this study and the study of Do and Faff, the main conclusion to be drawn is the existence of the positive relationship between profitability and the volatile period. In another article, Zhang (2012) considered the effect of different thresholds for the Distance method and different periods in the formation period in stock selection to assess profitability of the pairs trading strategy and reported that the relationship between profitability and threshold depended on the date of formation period and market situation. Lindberg (2014) modeled the spread between the prices of two stocks using the Ornstein Uhlenbeck process, and excess return was achieved after applying opportunity cost and stop loss constraint. Song and Zhang (2013) explored this strategy with a dynamic planning approach and modeled the value function using the Ornstein Uhlenbeck process. He also demonstrated applicability of the method considering commission and stop loss limit. Broussard and Vaihekoski (2012) used Gatev's research to measure profitability of this strategy in Finland market using different weighting structures. Their results reflected enormous profitability in spite of a one-day lag after observing the entrance signal. Pizzutilo (2013) proposed strong proofs of pairs trading profitability by studying the Italian market and considering all of the short sale limitations, trading costs, interest cost, and liquidated warrant in pairs trading. In order to develop the Zhang Study, Huck (2013) tested the minimum Distance method on S&P500 to measure sensitivity of returns to changes of the formation period length. Bogomolov (2013) developed a new non-parametric method based on Renko-Kai limitations only by assuming significant invariance of statistical properties (volatility) of price difference of the pairs over time and showed positive performance of trades in the American and Australian markets. Haque and Haque (2014) developed a highly profitable model for the developing country of Bangladesh market. In his model, pair selection takes place through the Johanson's test, and the pair is modelled using the Vector Error Correction (VCE) model, and finally a pair trading takes place using the remaining of estimated model. Iran market as the developing country has not

captured much attention to this strategy due to lack of short sale permission in its market. Similar to Haque study, the present research can be considered a less common example of studies on pairs trading in the Iranian capital market. Huck and Afawubo (2015) tested performance and profitability of different pair selection methods for pairs trading (namely the minimum Distance method, Cointegration, and Dicky Fuller) on the S&P500 index. Results of the tests revealed poor performance of the minimum Distance method. The Dicky Fuller test did not bring about much profit after application of trading costs, and the Cointegration method yielded considerable profit after application of trading costs and risk factors. Caldas et al. (2016) compared and analyzed levels of performance and profitability of the pairs selection in the United States, Brazil, and Europe using the minimum Distance and Cointegration methods. Their findings indicated that the Cointegration method performed better in Brazil and Europe, whereas the minimum Distance method yielded more profit in the United States. Like Caldas study, for lead to the accurate conclusion and be certain about study the global portfolio with covering all the aspect and situation, this research conducted on the both developing and developed countries including Iran and USA. Li et al. (2014) assessed performance of the pairs trading strategy by applying the Error Correction method to the firms dual listed on A-share market in China and H-share market in Hong Kong based on the Sharpe ratio and Value at Risk (VaR). Their results showed achievement of an annual excess return of 17.6%. Huck (2015) used stocks of the S&P500 and Nikkei225 indices to test performance of pairs trading with different pair selection methods (i.e. the minimum Distance method, Cointegration method, and the Stationery method). The Cointegration method gave the best performance in both markets. Rad et al. (2016) examined performance of three pairs trading strategies (the Distance, Cointegration, and Copula method) in the American market considering trading costs. As a result, the Distance method gave the best performance. According all the mentioned researches that conducted to compare the performance of different selection methods, it can give a rise to significance of the risk of inappropriate pair selection. Hence, as develop the article of Huck and Afawubo (2015), we compared four selection methods to find the best method across different asset classes in global market. Krauss (2016) reviewed the pair trading literature and classified the methods into five general groups namely the minimum Distance, Cointegration, Time series, Stochastic control, and other methods.

With regards to above studies that were carried out in this subject, it can be seen that the main focus of all was on the unite asset class (stock) to achieve the best performances under various conditions, and their findings corroborate its profitability in all situations. While the aim of this research was to study a combination of stock and commodity classes in pairs trading simultaneously to assess its performances for the first time. Concerning the results, this strategy can be introduced as a profitable one to the investors. The remainder of this paper is structured as follows. Section two presents a detailed description of all of the four pair selection methods. A description of data set is in section three. Results of the experimental investigation are presented in section four, which consists of the four pair selection methods results, trading period results, risk-adjusted return calculations, and return results. Finally, in section five, the results are analysed and conclusion and recommendations are presented.

2. Pairs Selection

According to Krauss (2016) and its classification of pairs trading in five general groups, only the first and second classes were studied in this research. The first class contains the Distance method, as the first method used in this area, which has been always highly profitable according to previous studies. The second group is called the Cointegration group, which includes econometrics concepts and analysis. The Augmented Dicky Fuller test, Granger Causality test, and Linear Regression Analysis are members of this family. In addition to the minimum Distance group, the Augmented Dicky Fuller test, Granger Causality test, and Linear Regression, a new method known as the Correlated Remaining method, which was not discussed in any of the previous studies except for the study by Fabozzi and Markowitz (2011), referred to in the 'Equity Valuation and Portfolio Management' book. Therefore, this method was for the first time assessed by real data in this paper.

2.1 Distance Method

In this method, pair price spreads are calculated as sum of squared differences (SSD) between two series of normalized prices. According to (Goetzmann and Rouwenhorst 1998, Haque and Haque 2014), the desired pair select by finding the minimum sum of squares of differences between two series of normalized prices. Therefore, first the prices were converted into normal prices using the following formula.

$$p_{*it} = \frac{p_{it} - E(p_{it})}{\sigma_i} \quad (1)$$

Where P is the normalized price of asset i at time t , $E(p)$ is the expected price, and σ is the standard deviation of the asset price.

Afterwards, the sum of squared differences of pairs by using normalized prices is calculated as follows.

$$SSD_{i,j} = \sum_{t=1}^T (P_{*i,j} - P_{*j,t})^2 \quad (2)$$

Where, T denotes the number of trading days during the formation period (Broussard and Vaihkoski 2012). In fact, pairs should be formed per each asset by finding another asset that minimizes the sum of squared differences (SSD). Finally, the pairs with the least SSDs are candidates for trading.

2.2 Augmented Dicky Fuller and Granger Causality Methods

The Augmented Dicky Fuller test is used to check existence of a unit root in a time series sample to determine it's stationary. For pair selection in this method, the spread of two assets should have a constant mean and volatility over time, so any price deviation from the equilibrium state can be considered an opportunity for opening a position in trading period. Therefore, pairs should be stationary and the unit root null hypothesis ($H_0 : \phi = 1$) is assessed using the following equation.

$$\Delta Y_t = \alpha_0 + \alpha_1 t + \theta Y_{t-1} + \sum_{i=1}^p \gamma_i \Delta Y_{t-i} + u_t \quad (3)$$

In this equation, y_t is the price rate $\frac{y_{i,t}}{y_{j,t}}$, α_0 is a constant, α_1 is the time trend coefficient, and p is order of autoregressive process lags (Gujarati 2003, Vidyamurthy 2004). First, the assets rejecting the null hypothesis are added to the list, and then in order to check existence of a correlation between the pairs, Granger's Causality test is conducted (Baronyan et al. 2010).

For the Granger causality test the following equation is formed between the selected asset prices and is estimated with the least squares method.

$$y_t = c + \sum_{i=1}^p \alpha_i y_{t-i} + \sum_{i=1}^p \beta_i x_{t-i} + \mu_t \quad (4)$$

Afterwards, the F statistic is obtained for the null hypothesis by estimating the following equation (Gujarati 2003).

$$y_t = c + \sum_{i=1}^p \gamma_i y_{t-i} + \varepsilon_t \quad (5)$$

$$RSS_\mu = \sum_{t=1}^T \mu_t^2 \quad \text{&} \quad RSS_\varepsilon = \sum_{t=1}^T \varepsilon_t^2 \quad (6)$$

If the $s = \frac{RSS_\varepsilon - RSS_\mu}{\sqrt{T-2p-1}}$ (test statistic) is smaller than the critical value, the null hypothesis (i.e. $\beta_i = 0$) is not rejected and consequently x_t is the Granger Cause of y_t , (Baronyan et al. 2010).

2.3 Linear Regression Method

For Linear Regression as pair selection method, regression equations between the two by two asset's prices are formed as $y_t = \alpha + bx_t + \varepsilon$. Afterwards, equation parameters are estimated using OLS (ordinary least squares) method.

$$\hat{\alpha} = \bar{y} - \hat{b}\bar{x} \quad (7)$$

$$\hat{b} = \frac{\sum_{t=1}^T [(y_t - \bar{y})(x_t - \bar{x})]}{\sum_{t=1}^T [(x_t - \bar{x})]} \quad (8)$$

$$\bar{A} = \left(\frac{\sum_{t=1}^T A_t}{T} \right) \quad (9)$$

$$\bar{B} = \left(\frac{\sum_{t=1}^T B_t}{T} \right) \quad (10)$$

Therefore, the regression equation with an acceptable correlation coefficient, which is obtained by calculating the least mean square error and the largest adjusted R^2 (which reflects goodness factor of the model, and the larger this value, the higher the Linear Regression model's prediction), is selected for each assets (Gujarati 2003).

$$MSE = \frac{1}{n} \sum_{i=1}^n (\hat{y} - y_i)^2 \quad (11)$$

Finally, the final list is selected by conducting the Augmented Dicky fuller test on remaining of the selected regression equations to prevent false regression. In other words, if the time series variables are not stationary, there is no significant relationship between variables and the coefficients of determination and t-test statistic of coefficients are large, which may lead to wrong interpretations of the false regression (Fabozzi and Markowitz 2011, Vidyamurthy 2004).

2.4 Correlated Remaining Method

Spearman's correlation coefficient is defined as the coefficient of Pearson's correlation between ranked variables. For this pair selection method, first the price series of each asset is estimated based on time to find the remaining, and then the Spearman's rank of the remaining is calculated based on their values. Finally, The Spearman's correlation coefficient of all ranks is calculated follows.

$$\rho = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)} \quad (12)$$

Where, $d_i = x_i - y_i$ shows the difference between ranks x_t and y_t . As the correlation of remaining ranks increases, for each asset the highest ρ value is selected as the asset's pair in pairs trading (Fabozzi and Markowitz 2011).

3. Research Statistical Data

In this study, Commodities, components of TEDPIX and S&P500 stock exchanges were used. Prices data of February 2013 to May 2015 were extracted from registered commodities and companies. This interval was divided into five periods, and each of them consisted of a 12-month formation period and an eight-month trading period: First period: February 2013-September 2014; Second period: April 2013-November 2014; Third period: June 2013-January 2013; Fourth Period: August 2013-March 2015; October 2013-May 2015. Daily data was used and due to the difference between holidays of the Iranian and international markets, the dates used in this research were matched (official holidays in international markets were replaced by data of previous days that were holidays in Iran). In addition, international data was multiplied by a constant foreign exchange rate to obtain a single exchange rate.

A total of 41 assets were selected from different asset classes which its stocks were the most liquid in the international market and Iran stock exchange by excluding the banking and automobile industries. Finally, industries such as the oil and gas industry, precious metals industry, industrial metals, and agricultural products remained, as well as the commodity in each industry were also selected. The data used in this study include:

1. Company stock price: Stock prices were extracted from TSE system, which is available on the Website of Tehran Stock Exchange Technology Management Company as well as S&P500 stock information on Yahoo.
2. Commodity price: It was obtained from different websites such as the Iranian Commodity Stock Exchange and United States' Commodity Stock Exchange.

4. Experimental Results

In this section, portfolios formation and analysing portfolios during the trading period is described in detail.

4.1 Results of Pairs Selection Methods Analysis

Among all of the existing pairs of 41 assets analysed in five periods, the pairs resulted from the Distance method had the lowest SSD as shown in Table 1.

Table 1: Results of pair selection in the Distance method

Pairs	Selected Pairs	Industry	Selected Pairs	Industry	SSD
First Period					
1	ASLRF	Silver	PLG	Platinum	0.000000746
2	Gold	Gold	Rice	Agriculture	0.0000112
3	IMPUY	Platinum	Corn	Agriculture	0.0000215
Second Period					
1	ASLRF	Silver	Rice	Agriculture	0.00000203
2	Gold	Gold	Platinum	Platinum	0.0000336
3	RIBT	Agriculture	Iron	Iron	0.0000461
Third Period					
1	VFFIF	Agriculture	Silver	Silver	0.00000401
2	SSN	Oil and Gas	Magsal Agri- MAGS1 & Magsal Agri.-R- MAGX ¹	Agriculture	0.0000232
3	Copper	Copper	Mobarakeh Steel Company ²	Iron	0.0000293
Fourth period					
1	HMY	Gold	Gold	Gold	0.000000311
2	IMO	Oil and Gas	SCCO	Copper	0.0000209
3	National Iranian Copper Industries Co	Copper	GPSX1- Piranshahr S.-R	Agriculture	0.0000556
Fifth period					
1	PBR	Oil and Gas	VFFIF	Agriculture	0.0000000385
2	CWEI	Oil and Gas	RIBT	Agriculture	0.00000986
3	CMGHF	Agriculture	NSSMY	Iron	0.000105

As seen in Table 1, in the output of this model a number of selected pairs of similar industries or dissimilar industries were obtained. In addition, a combination of assets was observed in the selected pairs. Moreover, the Iranian stock exchange shares were observed in the resulting portfolios in the final periods.

¹ Holding company in the field of agriculture and animal husbandry

² The biggest steel producer in Middle East and Northern Africa

In the Augmented Dicky Fuller method and Granger's Causality method, first the price rate in each of the five periods was obtained using $\frac{y_i}{y_j}$ for each of the 40 assets. Afterwards, based on the aforementioned relations and equations, the Augmented Dicky Fuller test with the null hypothesis of existence of a unit root was conducted in each price series. With the selected significant model (with intercept, intercept and trend, and neither intercept nor trend) and the calculated suitable lag, pairs rejecting the null hypothesis or stationary pairs entered the next step. Granger's Causality test was then conducted on the pairs. Pairs that did not reject the null hypothesis (with test statistic smaller than the critical value) or pairs in which Granger's Causality was one-way or two-way were added to the list as shown in Table 2.

Table 2: Selected pairs based on Augmented Dicky Fuller Test & Granger Causality Test

Pairs	Selected Pairs	Industry	Selected Pairs	Industry
First Period				
1	ASLRF	Silver	WRN	Copper
2	RIBT	Agriculture	Magsal Agri- MAGS1 & Magsal Agri.-R- MAGX	Agriculture
3	Corn	Agriculture	Magsal Agri- MAGS1 & Magsal Agri.-R- MAGX	Agriculture
Second Period				
1	ASLRF	Silver	Platinum	Platinum
2	HMI	Gold	Gold	Gold
3	SCCO	Copper	Persian Gulf Petrochemical Industries Co. Investment ³	Oil and Gas
Third Period				
1	PLG	Platinum	Rice	Agriculture
2	Corn	Agriculture	Gold	Gold
3	SRGL	Gold	Wheat	Agriculture
Fourth period				
1	IMPUY	Platinum	MTGRF	Iron
2	Gold	Gold	Persian Gulf Petrochemical Industries Co. Investment	Oil and Gas
3	Iron	Iron	GPSX1- Piranshahr S.-R	Agriculture
Fifth period				
1	KIROY	Iron	Iron	Iron
2	MTGRF	Iron	GPSX1- Piranshahr S.-R	Agriculture
3	Persian Gulf Petrochemical Industries Co. Investment	Oil and Gas	Mobarakeh Steel Company	Iron

According to Table 3, $t < C - Value$ for all of the selected pairs which shows rejection of the null hypothesis about existence of a unit root. Hence, the data series under study is stationary. In addition, P-Value results reflect the likelihood of the results when the null hypothesis is approved, and based on the null hypothesis in this test, values smaller than alpha prove accuracy of results. The P-Value of all of the selected pairs in the Granger's Causality test is smaller than 0.05, which reflects not rejection of the null hypothesis about existence of a Granger Causality relationship between two paired price data series.

³ Iranian holding company in Petrochemical industry

Table 3: The Estimated Parameters of Augmented Dickey Fuller test and Granger Causality test

Augmented Dickey Fuller test					Granger Causality test		
Pairs	α	P-Value	Critical value	Statistics of t-test	P-Value	Statistics of -test f	Causality
First Period							
1	0.05	0.0206	-3.4265	-3.7528	0.0153	3.5389	one-sided
2	0.05	0.0467	-1.9420	-1.9718	0.0439	2.7382	two-sided
3	0.1	0.0728	-1.6159	-1.7709	0.0366	2.8781	
Second Period							
1	0.05	0.0107	-2.8720	-3.4308	0.0075	4.0692	one-sided
2	0.05	0.0265	-2.8720	-3.1162	0.0254	3.1551	two-sided
3	0.01	0.0002	-3.9923	-5.04511	0.0006	3.9328	
Third Period							
1	0.1	0.0541	-2.5724	-2.8395	0.0293	3.0465	one-sided
2	0.1	0.0770	-1.6159	-1.7441	0.0406	2.7982	one-sided
3	0.01	0.0013	-3.9923	-7.5789	0.05	2.5798	two-sided
Fourth period							
1	0.05	0.0115	-3.4264	-3.9461	0.0290	3.0549	one-sided
2	0.05	0.0203	-2.8720	-3.2161	0.0422	2.7687	one-sided
3	0.05	0.0169	-3.4265	-3.8196	0.0389	2.8306	one-sided
Fifth period							
1	0.05	0.0289	-3.4265	-3.4265	0.0109	3.7927	one-sided
2	0.05	0.0377	-2.8720	-2.9841	0.0353	2.9057	one-sided
3	0.05	0.0181	-2.8721	-3.2534	0.0160	3.5014	one-sided

In the Linear Regression method, the first step is to create 41*41 regression equations with a combination of assets in the form of pairs. Afterwards, equation parameters are estimated using OLS and for each asset the smallest MSE and the maximum adjusted coefficient of determination are selected. The resulting regression relationship should also be a two-way relationship between the selected pairs. Finally, the Augmented Dicky Fuller test is carried out on equation remaining. The final list that enumerates three pairs for formation of a portfolio in each of the five periods is presented in Table 4.

Table 4: Selected Pairs in Method based on Linear Regression

Pairs	Selected Pairs	Industry	Selected Pairs	Industry
First Period				
1	CWEI	Oil and Gas	RSGUF	Agriculture
2	IMPUY	Platinum	WRN	Gold
3	Gold	Gold	Silver	Silver
Second Period				
1	CWEI	Oil and Gas	VFFIF	Agriculture
2	IMPUY	Platinum	PLG	Platinum
3	WRN	Gold	GPSX1-Piranshahr S.-R	Agriculture
Third Period				
1	CMGHF	Agriculture	Corn	Agriculture
2	RSGUF	Agriculture	VFFIF	Agriculture
3	Copper	Copper	National Iranian Copper Industries Co	Copper
Fourth period				
1	HMY	Gold	Gold	Gold
2	MTGRF	Iron	Oil Industry Investment Co	Oil and Gas
3	Wheat	Agriculture	Rice	Agriculture
Fifth period				
1	ARG	Oil and Gas	PLG	Platinum
2	MTGRF	Iron	Oil Industry Investment Co	Oil and Gas
3	PBR	Oil and Gas	RMCF	Agriculture

P-Value and the t-test statistic were obtained for evaluating the significance of the estimated regression coefficients. As seen in Table 5, the P-Value for each equation is considerably smaller than the $\alpha = 0.05$, and the test statistic absolute value for all of the three pairs in each of the five time periods is larger than two, which shows significance of the coefficients. In addition, these results reflect rejection of the null hypothesis about existence of a unit root, hence approval of stationary of the data series.

Table 5: Results of regression in Method based on Linear Regression

Linear Regression					Augmented Dickey Fuller test				
Pairs	P-Value	Statistics of t-test	MSE	Augmented Coefficient of Determination (R^2)	P-Value	Critical value	α	Statistics of t-test	
First Period									
1	0	-34.45	0.046	0.815	0.0351	-2.8720	0.05	-3.0101	
2	0	29.60	0.007	0.764	0.0159	-2.8720	0.05	-3.2991	
3	0	70.62	0.573	0.949	0.0042	-2.5735	0.01	-2.8675	
Second Period									
1	0	37.03	0.008	0.835	0.0435	-2.8720	0.05	-2.9274	
2	0	14.59	0.007	0.439	0.0018	-3.4543	0.01	-3.9801	
3	0	-17.39	0.006	0.526	0.0044	-3.4543	0.01	-3.7166	
Third Period									
1	0	-26.19	0.0001	0.715	0.0001	-3.4543	0.01	-4.6863	
2	0	-28.39	0.012	0.747	0.0053	-2.5735	0.01	-2.7906	
3	0	18.24	0.005	0.549	0.0021	-3.4543	0.01	-3.9361	
Fourth period									
1	0	28.72	0.035	0.752	0.0008	-3.4543	0.01	-4.1909	
2	0	23.74	0.005	0.674	0.0005	-3.4543	0.01	-4.3289	
3	0	29.4	0.2	0.76	0.0282	-2.8720	0.05	-3.0939	
Fifth period									
1	0	-152.03	0.036	0.988	0.005	-3.4545	0.01	-4.3060	
2	0	19.97	0.596	0.596	0.0041	-2.5735	0.01	-2.8734	
3	0	16.18	0.491	0.491	0.0216	-2.8721	0.05	-3.1917	

In the Correlated Remaining method, for each asset a pair with the highest level of correlation is selected and three pairs with two-way correlations are added to the portfolio as part of the final list shown in Table 6.

Table 6: Results of Correlated Remaining Method

Pairs	Selected Pairs	Industry	Selected Pairs	Industry	P-Value	Statistics of t-test	R
First Period							
1	RMCF	Agriculture	GPSX1- Piranshahr S.-R	Agriculture	0	237.06	0.997
2	WRN	Gold	Copper	Copper	0	46.02	0.941
3	Gold	Gold	Silver	Silver	0	49.58	0.949
Second Period							
1	Gold	Gold	Silver	Silver	0	75.62	0.977
2	Wheat	Agriculture	Chadormalu mining & Industrial Co. ⁴	Iron	0	30.40	0.879
3	Rice	Agriculture	IRALCO-Iranian Aluminum Co ⁵	Aluminum	0	26.68	0.851
Third Period							
1	ASLRF	Silver	Oil Industry Investment Co	Oil and Gas	0	47.24	0.944
2	Gold	Gold	Silver	Silver	0	63.44	0.967

⁴ The main Iron Ore Concentrate producer for Iron Making in Iran

⁵ The company manufactures aluminum ingots

Pairs	Selected Pairs	Industry	Selected Pairs	Industry	P-Value	Statistics of t-test	R
3	Platinum	Platinum	National Iranian Copper Industries Co	Copper	0	486.2	0.999
Fourth Period							
1	MTGRF	Iron	Oil Industry Investment Co	Oil and Gas	0	3231.5	0.999
2	NSSMY	Iron	Copper	Copper	0	1.35	0.904
3	Corn	Agriculture	Wheat	Agriculture	0	36.67	0.920
Fifth Period							
1	ARG	Oil and Gas	PLG	Platinum	0	315.46	0.998
2	VFFIF	Agriculture	Platinum	Platinum	0	104.61	0.987
3	Corn	Agriculture	Wheat	Agriculture	0	32.10	0.890

As seen in Table 6, very small values of P-Value (approximately zero) for the selected pairs reflect likelihood of approval of the null hypothesis about a zero correlation coefficient. These values also highlight existence of correlation among the pairs.

4.2 Analysis of Trading Period Results and Return Calculations

μ_t is the moving average and s_t is the historic moving standard deviation of the spread between prices over time. As explained by Gatev et al. (2006), the two standard deviation rule is used to start pairs trading. In fact, when the spread between two prices increases (decreases) and exceeds the limits of the two standard deviation, the position is opened that one has to buy asset at a lower price, and at higher prices the short sale position is taken. When the spread between two prices reaches the moving average, all the positions are re-closed that the longed asset should be sold, and the sold asset should be bought.

In this eight-month period, the two standard deviation rule is applied to each of the pairs in the portfolio. Therefore, if the selected pair price spread exceeds the $\mu \pm 2\sigma$ limit, the trade should be started by buying the asset at a lower price and selling it at a higher price using the short sale technique. When the price spread reaches the average price in the $\pm 0.5\sigma$ range, the trade should be concluded by selling the bought asset and buying the asset that was previously sold using the short sale technique. There are exceptions to this rule. For example, we can start a trade and no trade conclusion opportunity is found by the end of the 8th month, or the spread between prices may exceed the limits in the beginning of the eight-month period. The solution for the former involves ending the trade on the last day of the 8th month at current prices. To solve the latter, the spread should be overlooked until entering the range and taking the first position after exiting the range. Finally, depending on the number of starting and ending trades, the related return is obtained and the portfolio is calculated either. Li et al. (2014) used Sharpe ratio to show the risk-adjusted return of their research. The Sharpe ratio was calculated to measure the exposed risk on the portfolios during execution of the pairs trading strategy. Tables 7, 8, 9 and 10 present Sharpe ratios for each of the pair selection methods.

Table 7: The results of trading period in Distance method

Period	Pairs	Yield	Distance Method
1	1	0.23	
	2	0	
	3	0.13	
	Portfolio	0.12	
2	1	0.19	
	2	0.18	
	3	0.22	
	Portfolio	0.2	
3	1	0.14	
	2	0.23	
	3	0.22	
	Portfolio	0.2	

4	1 2 3 Portfolio	0.18 0 0.06 0.08			
5	1 2 3 Portfolio	0.1 0.13 0.21 0.15			
Overview	Average of yields	Standard Deviation (6)	Statistics of t-test	P-Value	Sharpe Ratio
TEDPIX	-0.0811	0.02302	-11.1963	0.0004	---
S&P500	0.0004	0.00007	-6.7503	0.0001	---
Portfolio	0.1522	0.0225	---	---	6.5867

Table 8: The results of trading period in Augmented Dickey Fuller and Granger

Period	Pairs	Yield	Augmented Dickey Fuller and Granger Causality test		
1	1 2 3 Portfolio	0 0.24 -0.17 0.02			
2	1 2 3 Portfolio	0 0.16 0.03 0.06			
3	1 2 3 Portfolio	0 -0.1 0 -0.03			
4	1 2 3 Portfolio	0.06 0 0 0.02			
5	1 2 3 Portfolio	-0.08 0.01 0.07 -0.001			
Overview	Average of yields	Standard Deviation (6)			
TEDPIX	-0.0811	0.0230	-3.2508	0.0314	---
S & P500	0.0004	0.00007	-0.8815	0.4037	---
Portfolio	0.0147	0.0162	---	---	0.6604

Table 9: The results of trading period in method based on Linear Regression

Period	Pairs	Yield	Method base on Linear Regression		
1	1 2 3 Portfolio	0 0.009 -0.003 0.02			
2	1 2 3 Portfolio	0 0.48 0.11 0.2			
3	1 2 3 Portfolio	-0.11 0.09 0.16 -0.05			

4	1 2 3 Portfolio	0.005 -0.07 0.12 0.02			
5	1 2 3 Portfolio	0.19 0.001- 0.07 -0.001			
Overview	Average of yields	Standard Deviation (6)	Statistics of t-test	P-Value	Sharpe Ratio
TEDPIX	-0.0811	0.0230	-4.0218	0.0158	---
S&P500	0.0004	0.00007	-1.8312	0.1044	---
Portfolio	0.0652	0.0354	---	---	1.7288

Table 10: The results of trading period in Correlated Remaining Method

Period	Pairs	Yield	Correlated Remainders Method		
1	1	0.14			
	2	-0.07			
	3	-0.003			
	Portfolio	0.02			
2	1	0.05			
	2	0			
	3	0.02-			
	Portfolio	-0.006			
3	1	0			
	2	-0.01			
	3	0.18			
	Portfolio	0.06			
4	1	0.08			
	2	-0.07			
	3	0.05			
	Portfolio	0.02			
5	1	0.19			
	2	0.009-			
	3	0.05			
	Portfolio	0.054			
Overview	Average of yields	Standard Deviation (6)	Statistics of t-test	P-Value	Sharpe Ratio
TEDPIX	-0.0811	0.0230	-4.3420	0.0122	---
S&P500	0.0004	0.00007	-3.1292	0.0140	---
Portfolio	0.0316	0.0099	---	---	2.7878

To evaluate the statistical significance of difference between the return resulted from the pairs trading method and the market return, the paired sample t-test was carried out that is a parametric test in which the mean of a united sample is calculated in two states. Paired sample t-test is used in 'before-after' studies, or when the samples are the matched pairs, or when it is a case-control study. Lack of statistical difference between mean of two paired samples is the null hypothesis. In fact, the opposite of this hypothesis is existence of a difference between means. The hypothesis about equality of two correlated mean is tested as follows:

$$H_0 : \mu_2 - \mu_1 = 0 \rightarrow H_0 : \mu_d = 0 \quad (13)$$

$$H_1 : \mu_2 - \mu_1 \neq 0 \rightarrow H_1 : \mu_d \neq 0 \quad (14)$$

In this study, the TEDPIX index's return and S&P 500 index's return was calculated in five periods, and paired sample t-test was carried out on the market returns and portfolio returns in five periods with a confidence interval of 95% for each of the four selected methods. Tables 7, 8, 9 and 10 present the result for each of the pairs selection methods.

According to Table (7), the second and third periods yielded the best returns using the distance method. Moreover, there is a significant difference between the average index returns and average portfolio return due to the large deviation of the P-value from 0.05. Hence, the hypothesis about equality of averages is rejected, which proves existence of a significant difference between the market return and the return resulted from the pairs trading strategy (which is higher).

As seen in Table 8, the second and third periods yielded the highest and lowest returns, respectively. Moreover, the asset combinations (which included commodities) did not result in considerable results. It is worth mentioning that considering deviation of the P-Value from 0.05 for TEDPIX index, the averages equality hypothesis is rejected and there is a significant difference, but in this model there is a smaller difference between the two returns than the previous model. However, for S&P 500 index due to the value of P-value which is greater than 0.05, the zero hypothesis is not rejected and there is not a significant difference.

According to the results of the linear regression analysis presented in Table 9, the second period offered the best result whereas the first period yielded the worst result. In addition, asset combinations delivered acceptable results. Similar to the previous methods, despite the less than 0.05 value of P-Value for TEDPIX index and rejecting the zero hypothesis, which is show the significant differences between two returns, there is not a significant difference between the two returns of portfolio and S&P 500 index considering the P-Value.

In the correlated remaining method (Table 10), the third period produced the best result and the second period yielded the worst result. There is also a significant difference between the two return values similar to previous methods.

Concerning the Sharpe ratio results, an acceptable Sharpe ratio is obtained according to tables 7, 8, 9 and 10 and a 0.6% risk-free annual return on 12-month treasury notes without coupons (which is relatively 0.4% for eight months) as well as the average return resulted from the pairs trading strategy with each of the selection methods. Therefore, the lowest levels of the risk-adjusted return were offered by models one, three, two, and four in the order mentioned. In other words, investment with the aforementioned strategies will be highly efficient and risk-free for foreign investors.

However, in the case of local investors, all methods except for the Distance method yielded returns higher than the market return but their returns are lower with a large deviation from the 12% risk-free return over eight months. In other words, investment in government bonds or deposits in Iranian banks offers higher risk-free returns.

5. Conclusion

The four methods tested for pair selection during the formation period and the selected pairs were tested during trading period to identify arbitrage opportunities with different asset classes combination in the Iran Stock Exchange, New York Stock Exchange, and Commodity Market. Finally, based on the studies by (Huck and Afawubo 2015, Caldas et al. 2016), using return and risk results obtained from each method, the methods were compared and the best one was chosen. To determine existence or lack of a significant difference between the selected portfolio return and market return, results of the paired sample t-test on portfolios were assessed and a significant difference was observed between the returns. Hence, the results were acceptable as compared to the market return and approved the research objectives.

In view of the average return of each method, the Distance method delivered the best result across different asset combination in pair trading strategy. After the Distance method, third, fourth, and second methods yielded the highest returns, respectively. In addition, to measure the risk-adjusted return of each selection method, the Sharpe ratio of each portfolio was calculated, and results suggested that the distance method yielded the highest Sharpe ratio, which is indicative of the lowest risk-adjusted return level. Third, second, and fourth methods had the highest Sharpe ratio and lowest risk-adjusted return, respectively. Hence, the Distance method can be considered the best method for application of the pairs trading strategy with different asset classes combination in global market. The

results of study conducted by Caldas et al. (2016) who showed the Distance method was the best method for the American capital market with an annual return of 10.47%. By achieving a monthly return of 0.38% and study by Rad et al. (2016) who introduced the Distance method as the best method for the American market, give authenticity to our results.

Pairs trading is a highly profitable market neutral strategy which is not a risk-free method. One of the risks associated with this strategy, results from wrong and inappropriate selected pairs. Based on the studies by Zhang (2012), economic conditions during the formation period have effects on the relationship between profitability and threshold rules. And these findings resulted of the period of recession in the Iranian capital market. Hence, it should be noted that this conclusion was obtained based on a case study in a limited and recession period on the Stock Market within numerous constraints (such as impossibility of short sale, impossibility of international trades in the Iranian capital market, ignoring liquidity of asset classes and cost of trading and commodity storage) and some simplifying hypotheses (such as assuming an equal weight for all assets in the beginning of trades). Therefore, this conclusion cannot be generalized to all pair trading and all periods.

In the following several suggestions are provided for future research. Since a pairs trading may deliver a negative return in some cases and periods, application of constraints to prevent an increase in the loss of a trade will positively influence results. Moreover, this comparison can be tested on other selection methods (such as mean reverting Gaussian Markov chain and Vasicek model) in future research. It is also recommended to test these methods with exert commission and trading cost to make the result more accurate.

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